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as correct as those of Greenwich, they would, of course, give Right Ascensions of equal exactness, while errors in declinations would be less from 20° to 50°, and above that somewhat greater. It is with hesitation that I give this estimate, because such forecasts are not of much value, and are open to the objection of seeming to claim credit for a status that has hereafter to be justified. My purpose is very different from that; it is to show that *prima facie* the almucantar has claim to be admitted of as great or greater precision than the transit circle, though over a somewhat more restricted range. It is not to be expected that it will prove an exception to invariable experience with new instruments, and yield results of the first rank of accuracy, without the cost of much thought and labour, although Mr. Chandler's early success with it is encouraging.

It is my intention in the first instance to test its performances by re-observing many times the transits of *Nautical Almanac* stars, of which 121 cross our circle of observation, with the intention of doing everything to increase, if possible, the fidelity of such fundamental points. Even if experience should hereafter suggest some other line as more profitable, I think time would be gained on the whole by first establishing thoroughly the character of the instrument and finding out its peculiarities by such a research.

The Effects of Stellar Rotation upon Spectrum Lines.

By A. Fowler.

The possible modification of the lines in a stellar spectrum by a rapid rotation of the star was first pointed out by Sir William Abney in 1877.* Provided that the axis is not directed towards the Earth, it is evident that the effect of rotation will be to broaden all the lines of the spectrum, since rays proceeding from the central meridian of the star disc will suffer no displacement on account of this motion, while those from the approaching and receding portions will be displaced by varying amounts towards the more and less refrangible ends of the spectrum respectively. Abney suggested that the finer lines of a spectrum might in this way be caused to disappear, and that the velocity of rotation might be derived from the observed amount of broadening. In the same year Vogel investigated the distribution of intensity in a line broadened in this manner,† but spectra satisfying the conditions were then unknown.

Many spectra in which all the lines are wide and ill defined have since been recorded, and in the case of α *Aquilæ* rotation has been adopted by Pickering, Lockyer, and Vogel as

* *Monthly Notices*, vol. xxxvii. p. 278.
† *Ast. Nach.* Bd. 90, p. 71.

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providing a simple and sufficient explanation of the observed appearance of the spectrum.

In view of the possible importance of such effects in the interpretation of other peculiar spectra, it has been thought desirable to investigate more completely the conditions relating to the modifications of lines by stellar rotation, and the results of this inquiry are given in the present paper.

The displacement of a line, or of any portion of a broadened line, is in all cases proportional to the component velocity in the line of sight of the vapour producing it. The intensity of a line broadened by rotation at any point in its breadth will therefore be determined by the proportionate amount of vapour having corresponding component velocity.

Loci of Points of Equal Component Velocity.

(1) The component velocity in the line of sight of any point on or within a rotating rigid sphere is proportional to the perpendicular distance of the point from a plane containing the axis and the Earth. The component velocity is therefore the same for all points which lie in a plane perpendicular to the equator of the star and passing through the Earth.

Thus, in fig. 1 *a*, where the axis is perpendicular to the line of sight, the component velocities in the line of sight are equal for all points which lie along the circle ABD, or in its plane, and are represented by the distance CB if the equatorial velocity be represented by CE.

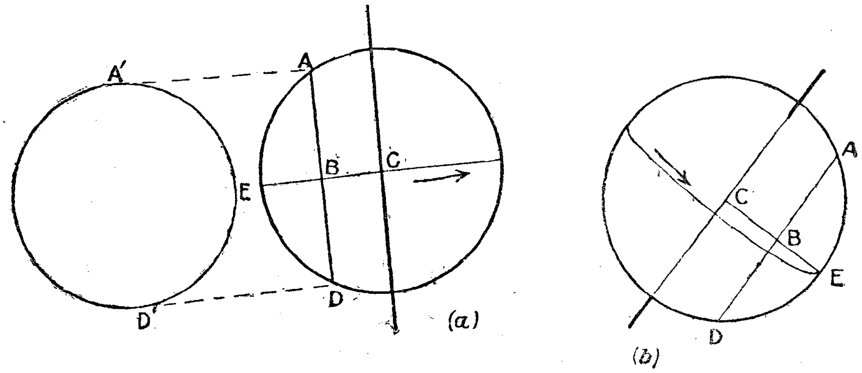

FIG. 1.—Illustrating circles of equal component velocities of rotating spheres.

When the axis is not perpendicular to the line of sight, as in fig. 1 *b*, the result is the same except that all the component velocities in the line of sight are reduced, the constant factor being the sine of the angle of inclination. If the axis be directed towards the Earth, the component velocities are zero, and there will be no broadening of the spectrum lines due to rotation.

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Effects of Rotation upon Dark Lines.

(2) If the absorbing vapours all move with the same angular velocity, the amount of broadening of the lines will be independent of the distance above the photosphere of the vapours which produce them, but will in all cases be equal to that due to the component velocities of a layer close to the photosphere.

This follows from (1), since only the vapours lying between us and the photosphere are effective in producing dark lines.

(3) Assuming that a star appears as a uniformly illuminated disc, and that its spectrum is the same in all parts of the disc, the intensity at any point of a broadened absorption line is proportional to the diameter of the circle of equal and corresponding component velocity.

The intensity curve may be constructed by taking distances CB (fig. 1) as abscissæ and AD as ordinates, and will be a semicircle if the intensity at the middle of the line is represented by a length equal to that taken to represent the greatest displacement to either side. The numerical values may be easily calculated, and are as follows :—

Distance from Middle of Line.	Intensity.	Distance from Middle of Line.	Intensity.
± 0·0	1·00	± 0·6	0·80
0·1	0·995	0·7	0·71
0·2	0·98	0·8	0·60
0·3	0·95	0·9	0·44
0·4	0·92	± 1·0	0·00
± 0·5	0·87	...	...

On account of the rapid fall in intensity near the extreme edges, the amount of broadening actually observable will necessarily be somewhat smaller than that due to the equatorial velocity.

If there be any considerable absorption near the edges of the star disc the lines will fade away more rapidly.

(4) The displacement corresponding to the greatest component velocity observable is equal to half the difference in breadth of a broadened line and the breadth of the same line when not affected by rotation.

The finer lines will thus appear to be more widened than the thicker ones. Thus, if a particular line is widened to three times its own thickness, a line of half the normal thickness will be widened to five times its own breadth.

(5) Lines, such as those of hydrogen in a *Sirian* star, which fade off at the edges when not affected by rotation, will not have the same distribution of intensity as the more common type of lines.

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Such a line may be regarded as built up of a number of components, each of which will be spread out into a broad band by the effect of rotation, and the ultimate form of the whole will be due to the integrated effects. The result will vary in individual cases, but in general the feeble edges of the lines will probably be so spread out that they will be ineffective in the final result ; it may be expected, therefore, that the effect of rotation on such lines will be to make the lines less diffuse on the edges and perhaps not much wider than the corresponding line in a similar star which is unaffected by rotation. (This appears to be the case in α *Aquilæ*.)

(6) The effects so far described will be modified if the absorbing vapours move with different angular velocities in different latitudes, as in the case of the Sun.

In the Sun, according to the spectroscopic measurements of Dunér, the angular velocities of the absorbing vapours decrease regularly from the Equator to the Poles, so that the proportion of particles of low velocity will be increased as compared with a star in which the angular velocity is constant. There will accordingly be a greater relative intensity towards the middles of the broadened lines.

The loci of points of equal velocity in the line of sight will no longer be chords perpendicular to the Equator, but will be such that in projection they appear as in fig. 2, which is drawn to scale in accordance with Dunér's observations. The ordinates of the intensity curves of the lines will be proportional to the lengths of *cd*, *ef*, &c., and the abscissæ will be respectively proportional to the distances *xy*, *xz*, &c.

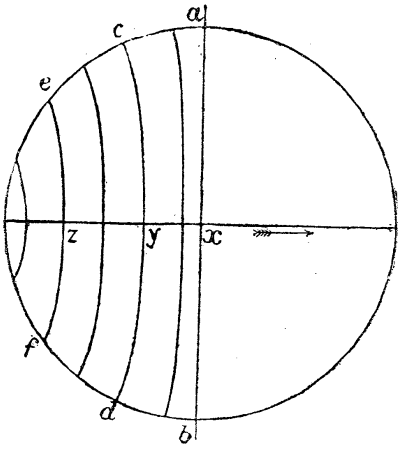

Fig. 2.—Loci of points of equal component velocities in the case of the Sun.

Effects upon Bright Lines.

In the case of bright lines the effects will generally differ from those produced in dark lines, but so little is yet known with certainty as to the conditions under which bright lines

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become visible in stellar spectra that the effects of rotation can only be investigated under assumed conditions. Still, some of the possible effects appear to be of sufficient interest to deserve mention. In each case the conclusions are based on the supposition that the gases producing bright lines are completely transparent.

(7) If the bright lines proceed from shells of incandescent vapours, the amount of broadening due to rotation will not necessarily be the same for all the lines.

Assuming the same angular velocity for all the vapours, it is evident that those extending furthest from the centre will have the greatest velocities, and that the lines which they give will be subject to the greatest amount of broadening.

(8) In the case of a spherical shell of vapour, no part of which is obstructed from view, the intensity of a bright line broadened by rotation will be the same in all parts corresponding to the component velocities of the inner surface, but will gradually diminish to zero at the edges, at points corresponding to the greatest component velocities of the outer surface.

In this case the intensity at any part of the line is determined by the sectional area of the ring of corresponding and equal component velocity. Thus, in fig. 3 the intensity at the point corre-

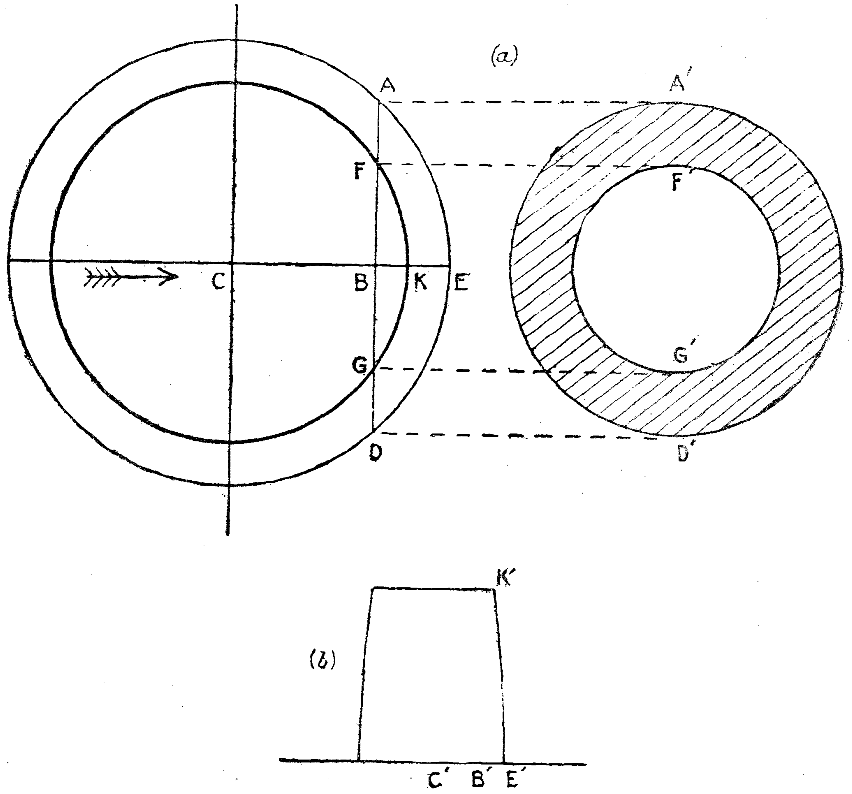

Fig. 3.—Derivation of intensity curve of a bright line produced by a complete shell of vapour, as modified by Rotation.

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sponding to the component velocity, CB, is proportional to the area of the circular ring, AFGD. Parallel sections of a hollow sphere have equal areas within the limits of the inner surface, and gradually diminish to the limit of the outer surface. Hence the intensity curve will be of the form shown in fig. 3 *b*.

(9) If the spherical shell of vapour which is assumed to produce a bright line is partly obstructed from view by a central spherical mass, there will be two maxima of intensity in the line when broadened by rotation.

As in previous cases, the points of equal component velocities will lie along circles in planes parallel to the axis of rotation and passing through the Earth, such as ABD in fig. 4 *a*, shown in projection in fig. 4 *b*.

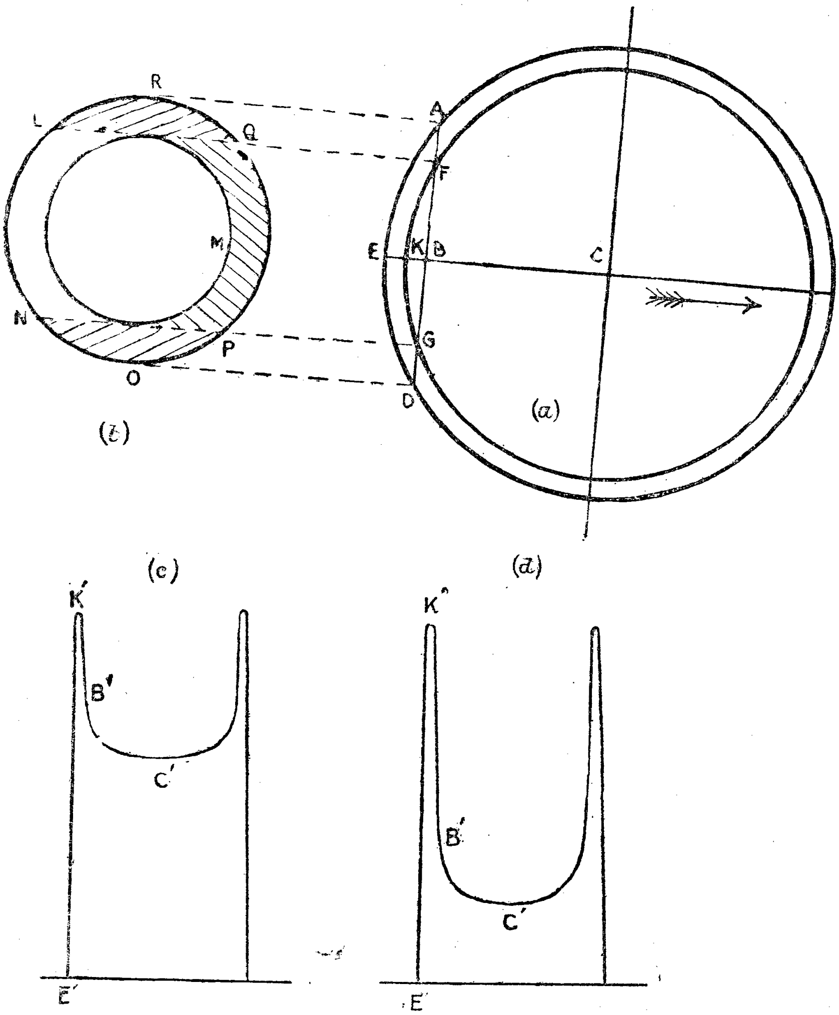

FIG. 4.—Derivation of intensity curves of bright lines, as modified by Rotation.

The intensity of a line at the point corresponding to the velocity, CB, will be proportional to the sectional area, LMNOPR,

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if the inner sphere has no effect on the vapour in front of it, and only obstructs the vapour behind.

The part of the line corresponding to the velocity CK will have an intensity proportional to the area of the circular section of the outer sphere at the point K. The resulting intensity curve is shown in fig. 4 *c*, which is drawn to scale for a case in which the radius of the inner surface is one-tenth less than that of the outer.

It is perhaps more nearly approaching the probable conditions in some of the stars to suppose that the vapour lying between us and the central mass is absorbing, so that the bright lines would only be produced by vapours round the limb. In that case the intensity of the part of a line corresponding to a given velocity, CB, would be represented by the combined areas of the segments LRQ and NOP in fig. 4 *b*. The intensity curve derived in this way for the hypothetical case in which the inner radius is one-tenth less than the outer is shown in fig. 4 *d*. It will be seen that in this case the two maxima are much more marked than in the previous one, and a distinct appearance of doubling would probably be observed. This result might be modified if the gas were not completely transparent; but since any absorption by external layers would be effective at all points of the limb, it does not seem likely that the modification would be such as to obliterate the double maximum.

Possible Applications.

In his most recent classification of stellar spectra Professor Pickering has given special prominence to the character of the lines; and from figures which he gives it appears that among stars of the *Sirian* and *Orion* types spectra with diffuse lines are not less numerous than those with sharp lines, while intermediate varieties are also plentiful. Thus, of the spectra included in the first ten of his groups, 71 have lines of ordinary sharpness, 91 have diffuse lines, while 113 are of intermediate character. It does not seem unreasonable to suppose that rotation is the cause of the diffuseness of the lines in many of those spectra where all the lines are affected, though it fails to explain the special broadening of the lines of helium in such a star as *ζ Tauri*. The varying degrees of broadening of the lines in different spectra may be brought about by different surface velocities, or may depend upon the directions of the axes of rotation.

The distribution of intensity in the broadened lines will depend to some extent upon the laws of variation of angular velocity in different stellar latitudes, as already explained.

As regards stars with bright lines, the data at present available are in most cases insufficient to permit an investigation of the presence or absence of rotation effects. In one class of stars, however—those like *γ Cassiopeiæ*—the result of an

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examination of the spectra from this point of view is most suggestive. In some of these stars the hydrogen lines are bright and double on a broad dark band, and it has been supposed that they are "double reversals," similar to those of calcium lines in the Sun. Pickering has found that in all cases where the hydrogen lines present this appearance the accompanying dark lines of the spectrum are very wide and feeble. Rapid rotation is therefore suggested by the appearance of the dark lines, and the foregoing investigation suggests that under these circumstances the bright lines would appear double, as we actually see them. There is a possible test as to whether in such stars we really see the effects of rotation—namely, that in stars like δ and μ *Centauri*, which resemble γ *Cassiopeiæ*, except that the dark lines are much less diffuse, the bright lines should either be single or only very closely double, whereas if they are "double reversals" their appearance would seem to be independent of the character of the dark lines. Dr. McClean informs me that his photographs of these spectra do not furnish conclusive evidence on this point, since the dispersion employed was insufficient to exhibit the doubling of the lines in γ *Cassiopeiæ*. It may be added that in γ *Cassiopeiæ* the separation of the components of the double bright lines is five or six times as great as that in the case of the doubly reversed K line in the Sun.

It does not seem improbable, therefore, that some of the points to which attention has been drawn may be found to have a useful bearing on the interpretation of some of the peculiarities which are met with in a survey of stellar spectra.

Inquiry as to the Cause of the Shadow Bands upon the Earth which accompany Total Eclipses of the Sun. By G. Johnstone Stoney, M.A., D.Sc., F.R.S.

If, when the Sun is shining, we look at a shallow pool on the sand of the sea-shore, there is usually a sufficient ripple over the surface of the water to give rise to a very beautiful optical effect. The appearance consists of alternate bands of intense brightness and of shade which seem to travel over the sand at the bottom of the water in harmony with the motion of the waves at the top.

When we consider the cause of this phenomenon, we can see that if there were a second Sun in the heavens so situated that the bright and dark bands it would produce on the sand would coincide with the dark and bright bands of the first Sun, then the two Suns operating together would tend to produce a uniform illumination over the sand, and the appearance of bands would be lost. It is in this way that on a cloudy day, when light arrives from all directions, there is no trace of the banded